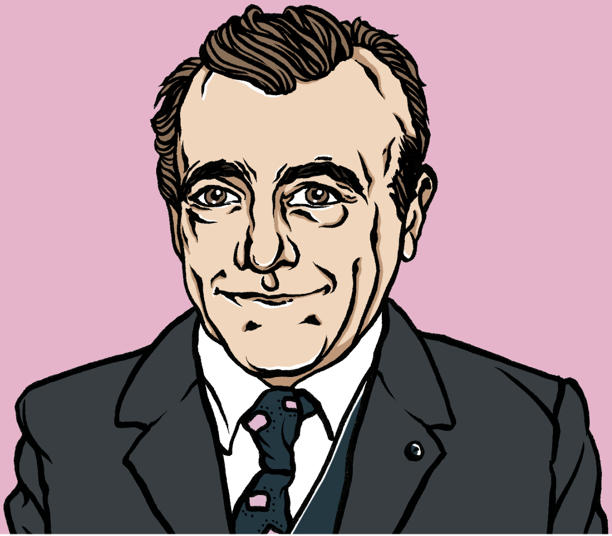
L'ANTIFERROMAGNÉTISME

— 1936 —

L'ANTIFERROMAGNÉTISME

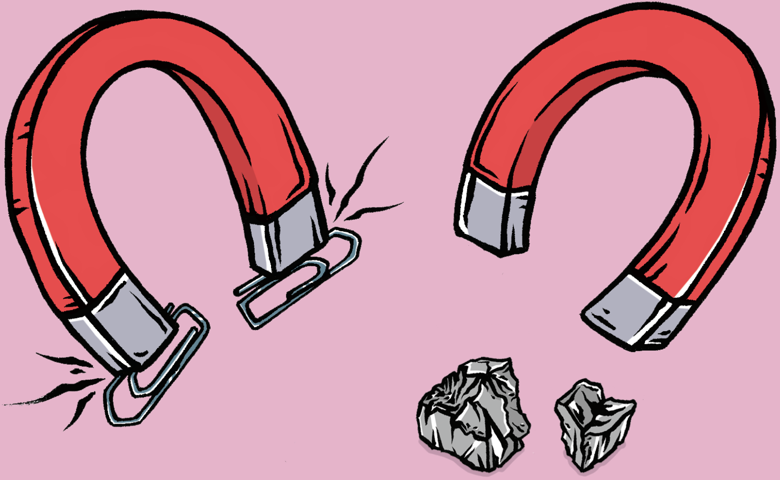
INSTITUT DE PHYSIQUE DE STRASBOURG, FRANCE

L'ANTIFERROMAGNÉTISME



L. NÉEL

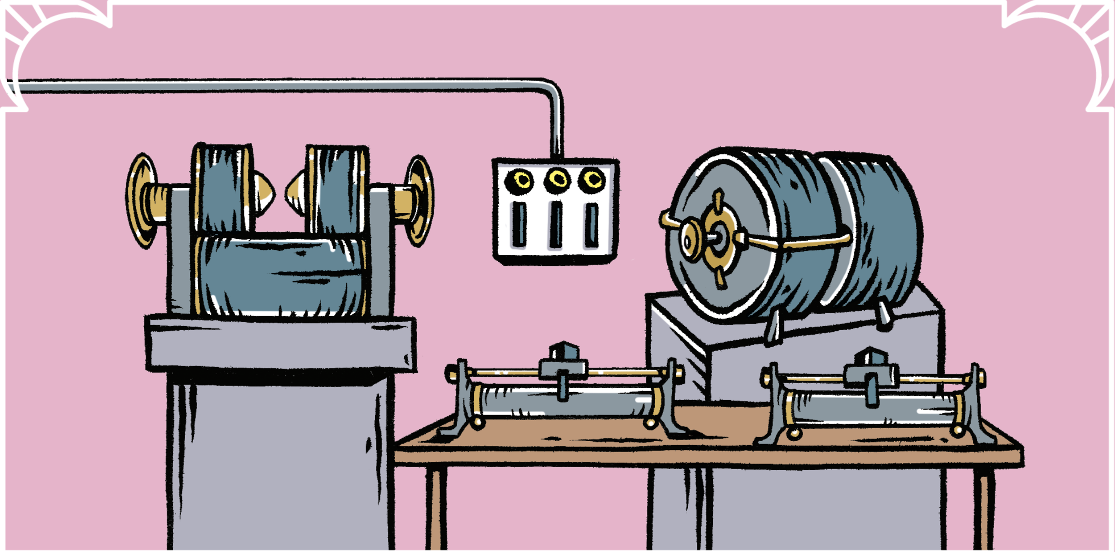
L'ANTIFERROMAGNÉTISME



LA QUESTION

Pourquoi certains métaux ou oxydes, par exemple le chrome, ne semblent pas magnétiques ?

L'ANTIFERROMAGNÉTISME



LE LABO

L'ANTIFERROMAGNÉTISME

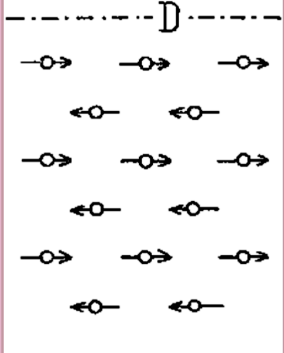
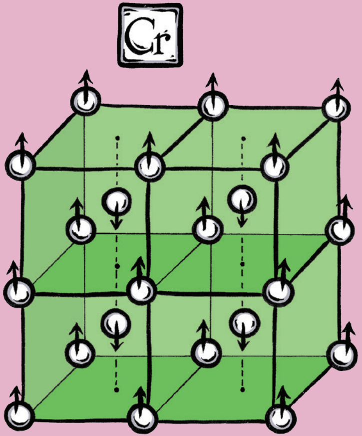


Fig. 8.



LE RÉSULTAT

Dans certains métaux et oxydes, les atomes portent de petits aimants, les spins, qui s'organisent tête-bêche. Ces matériaux appelés antiferromagnétiques ne présentent plus de pôles même si ils sont aussi ordonnés qu'un vrai aimant.

L'ANTIFERROMAGNÉTISME

PROPRIÉTÉS MAGNÉTIQUES DE L'ÉTAT MÉTALLIQUE ET ÉNERGIE D'INTERACTION ENTRE ATOMES MAGNÉTIQUES

Par M. Louis NÉEL

SOMMAIRE. — Une première partie du travail (§ 1 à 10) est consacrée à l'interprétation des expériences de M. Manders sur les variations, en fonction de la température, de la susceptibilité magnétique de quelques solutions solides à base de nickel (Ni et Al ou Ti, Sn, Sb, V, Mo, W, Cr). On étudie et on interprète les variations, en fonction du titre, de la constante de Curie et du coefficient de paramagnétisme constant superposé. On en déduit que les électrons magnétiques du nickel restent en nombre constant lorsqu'on passe de l'état ferro à l'état paramagnétique.

Dans une deuxième partie (§ 11 à 29), on expose comment on peut définir et calculer une énergie d'interaction magnétique entre deux atomes voisins porteurs de moment, à partir des données expérimentales, soit pour les ferromagnétiques, soit pour les corps à champ moléculaire négatif (Pd et Pt), soit pour les corps paramagnétiques à susceptibilité indépendante de la température (Mn, Cr, Ti, Mo, Ru, Rh, etc.). On étudie ensuite les variations de l'énergie d'interaction avec la distance entre les couches magnétiques des atomes interagissant et on montre qu'en première approximation l'énergie d'interaction ne dépend que de cette distance et varie très régulièrement avec elle. Cette conception permet d'interpréter et de relier entre eux un certain nombre de faits expérimentaux dont quelques-uns sont passés en revue.

Enfin, en supposant qu'il existe un couplage entre le réseau cristallin et les spin responsables du magnétisme des métaux, apparaissent des propriétés curieuses qui semblent être un point de départ pour expliquer les propriétés magnétiques compliquées du platine (§ 18, 19 et 30).

§ 16. **Calcul de w_{AB} d'après les données expérimentales.** — Si la concentration du métal B est petite, on a :

$$\theta' = PaC_A + Q\frac{b^2}{a}C_B \quad \text{et} \quad C' = PC_A + QC_B\left(2\frac{b}{a} - \frac{b^2}{a^2}\right) \qquad (11)$$

soit, en fonction du titre, une variation linéaire du point de Curie et de la constante de Curie apparente. Prolongeons les droites obtenues jusqu'à $Q = 1$; soit θ'' et C'' les valeurs de θ' et C' correspondant à $Q = 1$, d'après (11) on a :

$$\frac{C''}{\theta''} = \frac{2}{b} - \frac{1}{a} \quad \text{ou} \quad b = \frac{2}{\frac{C''}{\theta''} + \frac{C_A}{\theta_A}} \qquad (12)$$

en remarquant que pour le métal A pur, de constante de Curie C_A et de point de Curie θ_A, on a : $a = \frac{\theta_A}{C_A}$. C'' et θ'' se déterminent expérimentalement en extrapolant les tangentes initiales aux courbes de variation de la constante de Curie et du point de Curie en fonction du titre. J'ai appliqué cette méthode pour calculer les énergies d'interaction des liaisons mixtes w_{AB} : Ni-Co, Ni-Fe, Co-Fe, d'après les données expérimentales de Preuss (10), de Peschard (11) et de Bloch (14).

Dans le calcul précédent, w_{AB} représente l'énergie totale d'interaction entre deux moments μ_A et μ_B. Pour avoir des valeurs comparables aux w du § 14, il faut exprimer w_{AB} au moyen de l'énergie w'_{AB} d'interaction de deux électrons, portés l'un par l'atome A et l'autre par l'atome B. Posons $\mu_A = q\mu$, $\mu_B = q'\mu$ en désignant par μ le magnéton de Bohr. On a immédiatement : $w_{AB} = qq'w'_{AB}$. D'où, d'après la formule 8, puisque le facteur qq' disparaît haut et bas :

$$b = \frac{2pw'_{AB}}{N\mu^2}. \qquad (13)$$

Le tableau 5 donne les valeurs de C'', θ'', w'_{AB} correspondant à différentes liaisons. Le système cristallin étant le cube à faces centrées, on a toujours : $2p = 12$.

région où la formule 3 n'est pas valable, d'où la nécessité d'une étude spéciale de cette région qui sera pour les corps

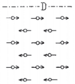


Fig. 8.

à champ moléculaire négatif la réplique de la région ferromagnétique des corps à champ moléculaire positif.

Au zéro absolu, chaque atome se dispose antiparallèlement à ses voisins, de manière à réaliser un assemblage d'énergie

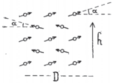


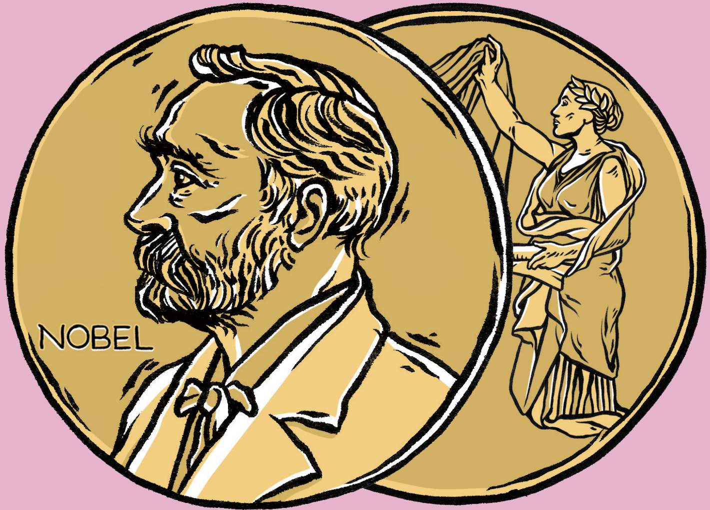
Fig. 9.

potentielle minimum comme celui qui est représenté sur la figure 8. Les moments sont tous parallèles à une même direction D, mais ils sont dirigés dans des sens différents au lieu d'être tous de même sens comme dans les ferromagnétiques. Un champ magnétique h, perpendiculaire à la direction D, va déformer cet assemblage et l'aimanter. Tous les

L'ARTICLE

Propriétés magnétiques de l'état métallique et énergie d'interaction entre atomes magnétiques, L. Néel, Annales de Physique, **5**, 232 (1936)

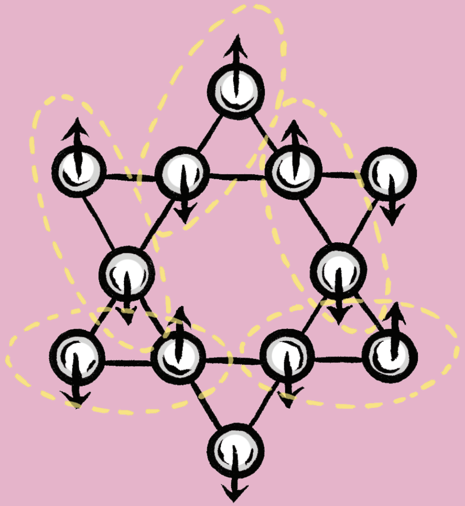
L'ANTIFERROMAGNÉTISME



L. NÉEL, PRIX NOBEL, 1970

Pour la découverte de l'antiferromagnétisme et du ferrimagnétisme qui ont mené à d'importantes applications en physique des solides.

L'ANTIFERROMAGNÉTISME



AUJOURD'HUI

De nouvelles formes d'aimants sont au cœur des recherches actuelles. Par exemple, dans les « liquides de spin », les spins placés en étoiles refusent de s'ordonner et se placent dans plusieurs états quantiques à la fois.

L'ANTIFERROMAGNÉTISME

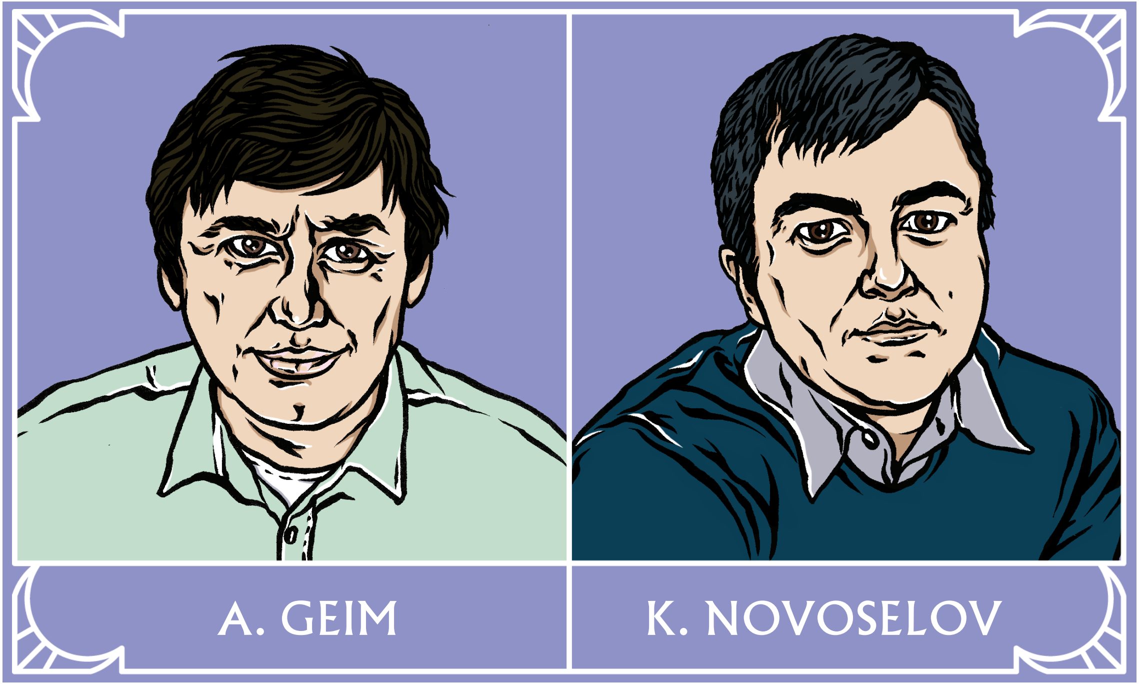
LE GRAPHÈNE

— 2004 —

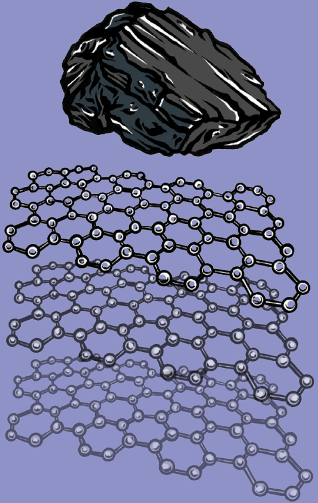
LE GRAPHÈNE

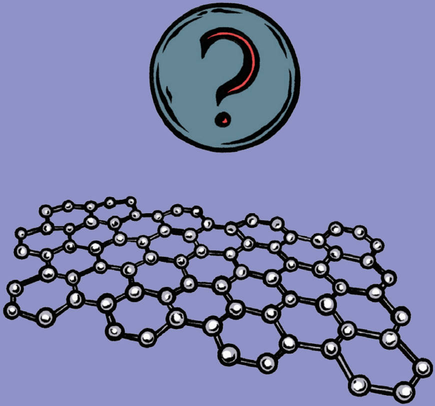
UNIVERSITÉ DE MANCHESTER, ANGLETERRE

LE GRAPHÈNE


A. GEIM
K. NOVOSELOV

LE GRAPHÈNE

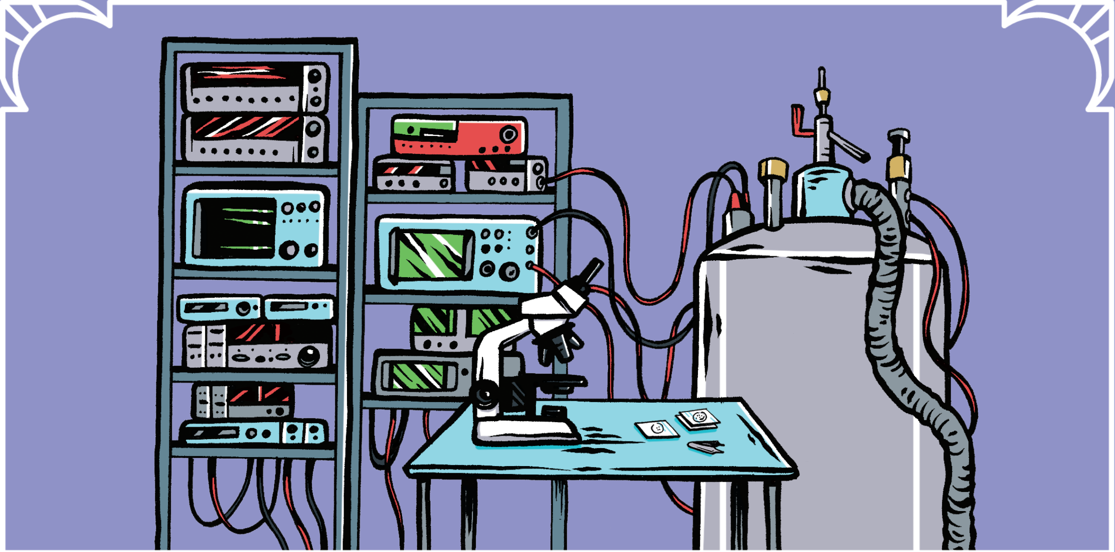




LA QUESTION

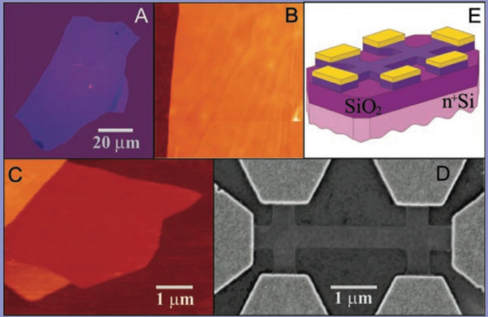
Peut-on fabriquer une couche d'un seul atome d'épaisseur à partir du charbon ?
Et quelles seraient alors ses propriétés ?

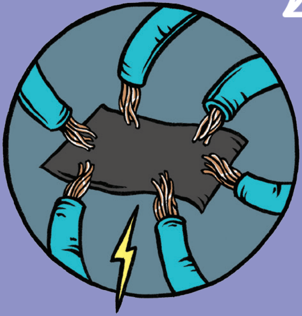
LE GRAPHÈNE



LE LABO

LE GRAPHÈNE





LE RÉSULTAT

On peut fabriquer, observer et mesurer une seule couche d'atomes de carbone, appelée graphène. Elle présente des propriétés électriques étonnantes, ni tout à fait métalliques, ni tout à fait isolantes.

LE GRAPHÈNE

L'ARTICLE

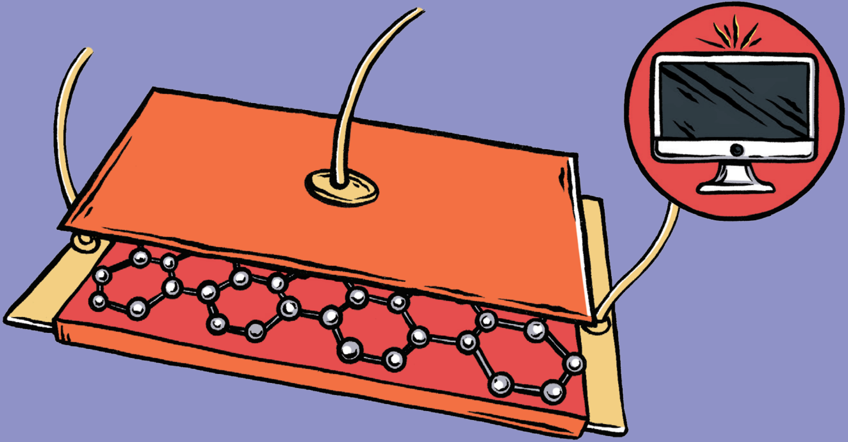
Electric Field Effect in Atomically Thin Carbon Films, K.S. Novoselov, et al., Science **306**, 666 (2004)

LE GRAPHÈNE

PRIX NOBEL, 2010

Pour des expériences révolutionnaires concernant le matériau à deux dimensions graphène.

LE GRAPHÈNE



AUJOURD'HUI

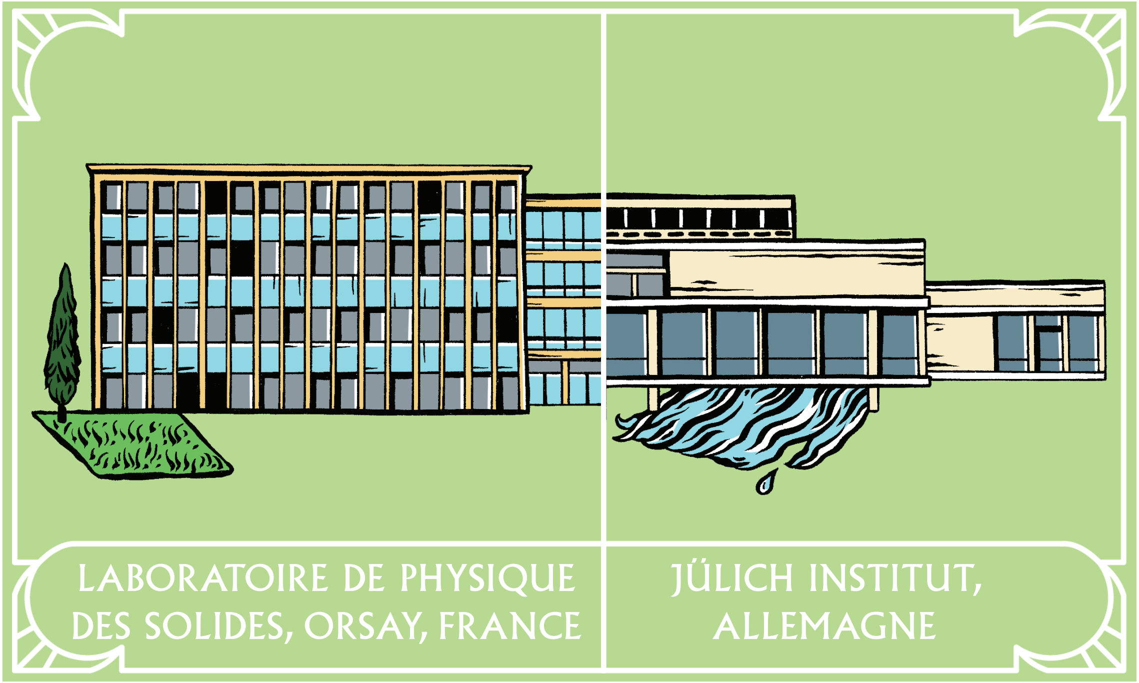
Le graphène pourrait avoir de nombreuses applications en particulier dans la nanophysique.
Il jouera peut-être un rôle essentiel dans l'électronique du futur.

LE GRAPHÈNE

LA MAGNÉTORÉSISTANCE GÉANTE

— 1988 —

LA MAGNÉTORÉSISTANCE GÉANTE

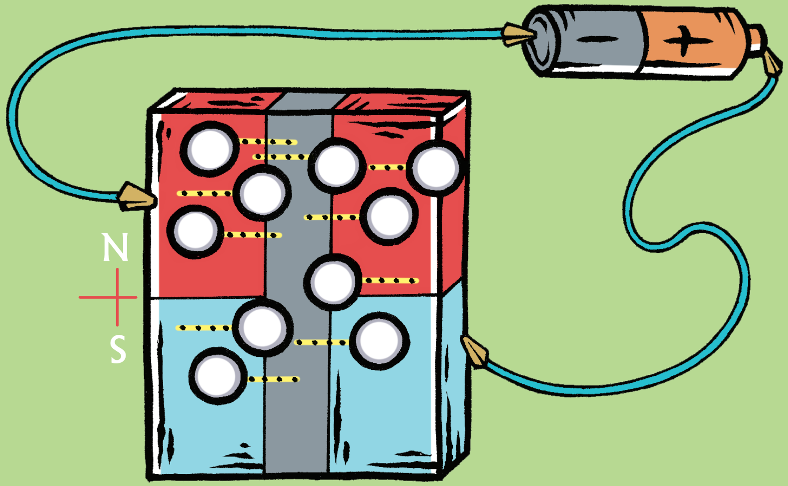

LABORATOIRE DE PHYSIQUE DES SOLIDES, ORSAY, FRANCE
JÜLICH INSTITUT, ALLEMAGNE

LA MAGNÉTORÉSISTANCE GÉANTE

A. FERT

P. GRÜNBERG

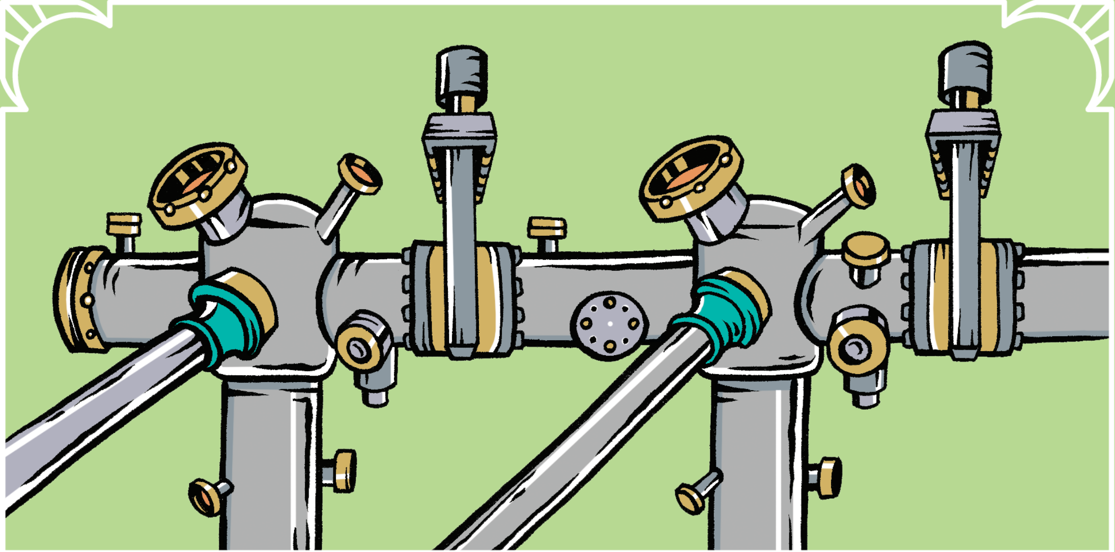
LA MAGNÉTORÉSISTANCE GÉANTE



LA QUESTION

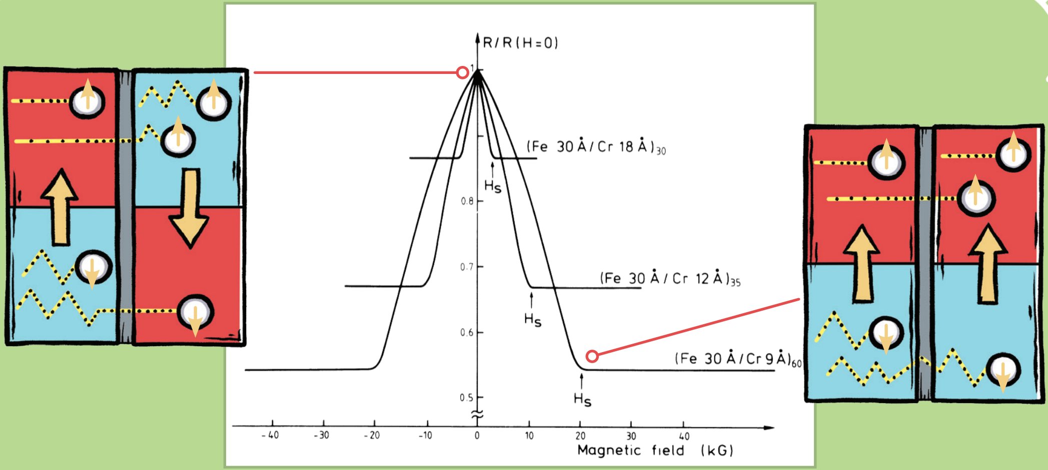
Le courant électrique dans de fines couches d'aimants est-il affecté par le sens de leurs pôles ?

LA MAGNÉTORÉSISTANCE GÉANTE



LE LABO

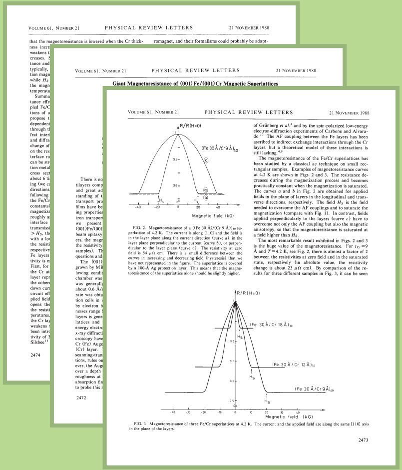
LA MAGNÉTORÉSISTANCE GÉANTE

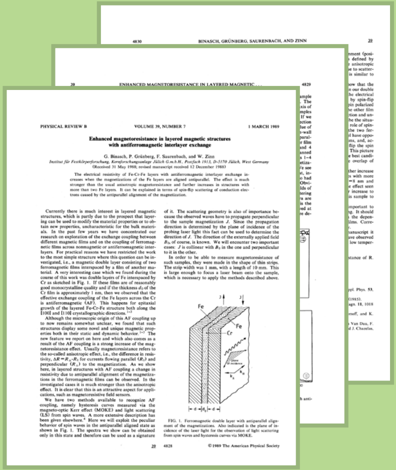


LE RÉSULTAT

Si on construit un « sandwich » magnétique et qu'on change ses pôles, sa résistance électrique varie énormément. En effet, les électrons possèdent eux aussi un petit aimant, le spin, qui interagit avec le sandwich magnétique.

LA MAGNÉTORÉSISTANCE GÉANTE





LES ARTICLES

Giant magnetoresistance of Cr magnetic superlattices, M. N. Baibich et al., PRL **61**, 2472 (1988)

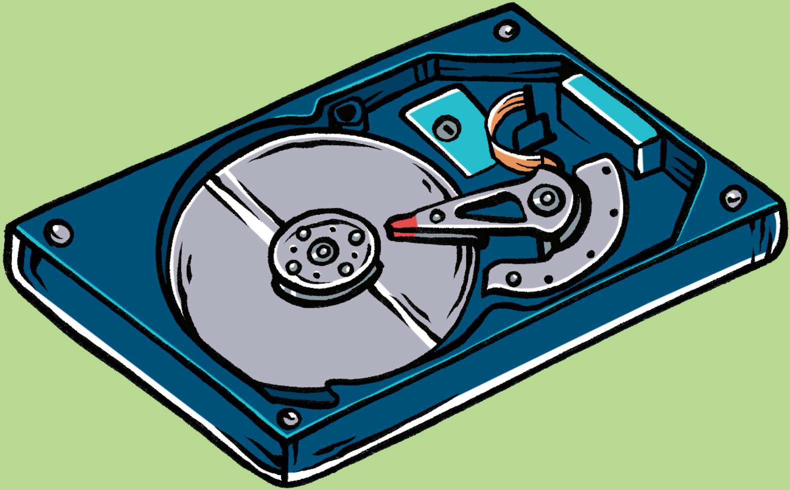
Enhanced magnetoresistance in layered magnetic structures, G. Binasch et al., PRB, 39, 4828 (1989)

LA MAGNÉTORÉSISTANCE GÉANTE

A. FERT, P. GRÜNBERG, PRIX NOBEL, 2007

Pour la découverte de la magnétorésistance géante.

LA MAGNÉTORÉSISTANCE GÉANTE



AUJOURD'HUI

Cette découverte a permis de développer les têtes de lecture des disques durs modernes.
Elle a aussi ouvert la voie à un nouveau champ de recherche : la spintronique.

LA MAGNÉTORÉSISTANCE GÉANTE

LA SUPRACONDUCTIVITÉ

— 1911 —

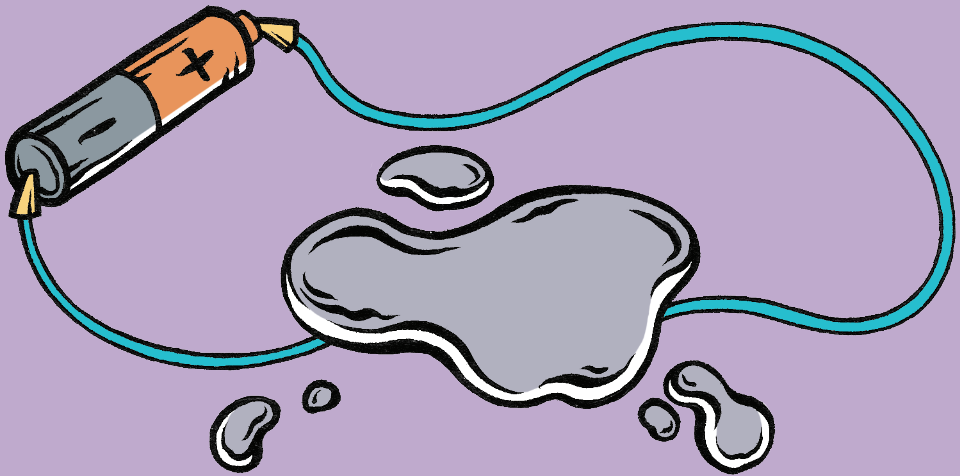
LA SUPRACONDUCTIVITÉ

UNIVERSITÉ DE LEYDE, PAYS-BAS

LA SUPRACONDUCTIVITÉ

K. ONNES

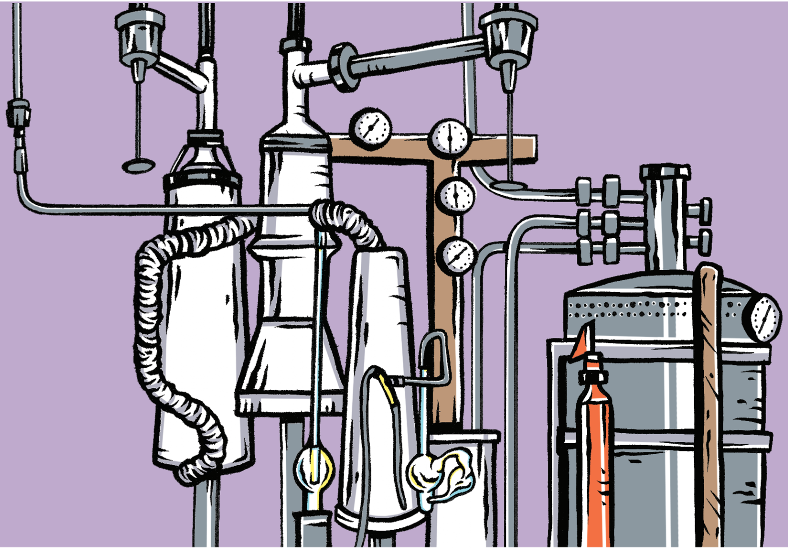
LA SUPRACONDUCTIVITÉ



LA QUESTION

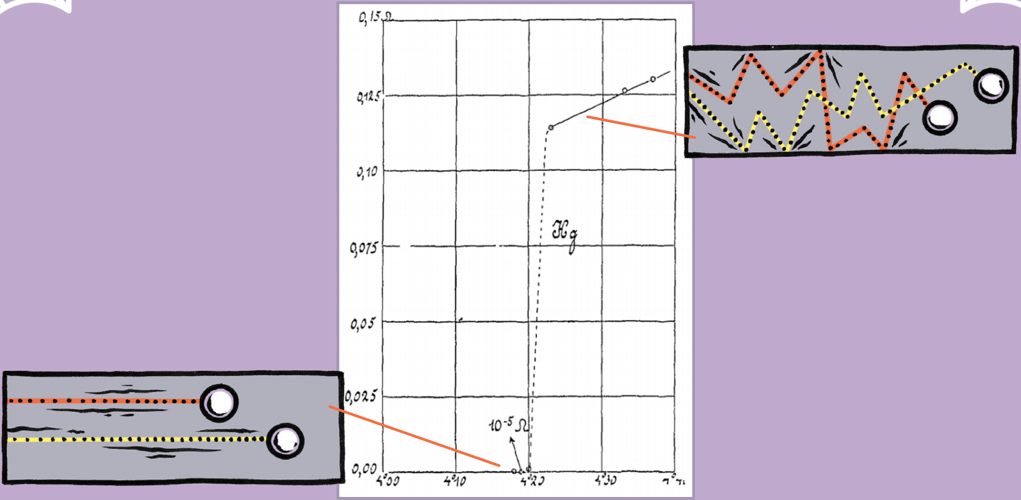
Un métal, ici le mercure, conduit-il mieux ou moins bien à basse température ?

LA SUPRACONDUCTIVITÉ



LE LABO

LA SUPRACONDUCTIVITÉ



LE RÉSULTAT

La résistance électrique du mercure chute brutalement à zéro à basse température.
Le métal conduit parfaitement : c'est la supraconductivité.

LA SUPRACONDUCTIVITÉ

decide, a theory of course which first of all takes account of the fundamental chemical facts which we mentioned above, but which further succeeds in avoiding the drawbacks — particularly with respect to the specific heats — which adhere to the hypothesis on the chemical forces sketched more at length in our previous paper. And then it cannot be doubtful, in our opinion, by what way we shall have to try to find such a theory. We shall have to extend the theory of indivisible units of energy, which has led to such remarkable results, to the chemical phenomena; it will be necessary to investigate in what way the properties of the reversible chemical reactions are connected with the phenomena of radiation. When this connection has been found, the course is indicated to calculate the difference of entropy of a chemical reaction by the aid of the statistical theory of entropy at temperatures at which this reaction can actually take place, and then it will be very simple to calculate by the aid of the acquired knowledge of the specific heats the difference of entropy also for temperatures, at which there can no longer be question of chemical reactions.

One of us has been occupied with this question, and hopes to be able before very long to publish further communications on this subject.

Physics. — "*Further Experiments with Liquid Helium. G. On the Electrical Resistance of Pure Metals, etc. VI. On the Sudden Change in the Rate at which the Resistance of Mercury Disappears.*" By H. KAMERLINGH ONNES. Communication N°. 124*c* from the Physical Laboratory at Leiden.

(Communicated in the meeting of November 25. 1911).

§ 1. *Introduction.* In Comm. N°. 122*b* (Proc. May 1911) I mentioned that just before this resistance disappeared practically altogether, its rate of diminution with falling temperature became much greater than that given by the formula of Comm. N°. 119. In the present paper a closer investigation is made of this phenomenon.

§ 2. *Arrangement of the resistance.* A description was given in Comm. N°. 123 (Proc. June 1911) of the cryostat which, by allowing the contained liquid to be stirred, enabled me to keep resistances at uniform well-defined temperatures; and in that paper I also mentioned that measurements of the resistance of mercury at the lowest possible temperatures had been repeated using a mercury resistance with mercury leads. The immersion of a resistance with such leads in a bath of liquid helium was rendered possible only by the successful construction of that cryostat.

The accompanying Plate, which should be compared with the Plate of Comm. N°. 123, shows the mercury resistance with a portion of the leads; it is represented diagrammatically in fig. 1. Seven glass *U*-tubes of about 0.005 sq. mm. cross section are joined together at their upper ends by inverted *Y*-pieces which are sealed off above, and are not quite filled with mercury; this gives the mercury an opportunity to contract or expand on freezing or liquefying without breaking the glass and without breaking the continuity of the mercury thread formed in the seven *U*-tubes. To the *Y*-pieces b_0 and b_8 are attached two leading tubes Hg_1, Hg_2 and Hg_3, Hg_4 (whose lower portions are shown at Hg_{10}, Hg_{20}, Hg_{30}, Hg_{40}) filled with mercury which, on solidification, forms four leads of solid mercury. To the connector b_4 is attached a single tube Hg_5, whose lower part is shown at Hg_{50}. At b_0 and b_8 current enters and leaves through the tubes Hg_1 and Hg_4; the tubes Hg_2 and Hg_3 can be used for the same purpose or also for determining the potential difference between the ends of the mercury thread. The mercury filled tube Hg_5 can be used for measuring the potential at the point b_4. To take up less space in the cryostat and to find room alongside the stirring pump *Sb*, the tubes which are shown in one plane in fig. 1 were closed together in the manner shown in fig. 2. The position in the cryostat is to be seen from fig. 4 where the other parts are indicated by the same letters as were used in the Plate of Comm. N°. 123. The leads project above the cover Sb_1 in a manner shown in perspective in fig. 3. They too are provided with expansion spaces, while in the bent side pieces are fused platinum wires Hg_1', Hg_2', Hg_3', Hg'_4, Hg_5' which are connected to the measuring apparatus. The apparatus was filled with mercury distilled over in vacuo at a temperature of 60° to 70° C. while the cold portion of the distilling apparatus was immersed in liquid air.

§ 3. *Results of the Measurements.* The junctions of the platinum wires with the copper leads of the measuring apparatus were protected as effectively as possible from temperature variation. The mercury resistance itself with the mercury leads, which served for the measurement of the fall of potential seemed, however, on immersion in liquid helium to be the seat of a considerable thermo-electric force in spite of the care taken to fill it with perfectly pure mercury. The magnitude of this thermo-electric effect did not change much when the resistance was immersed in liquid hydrogen or in liquid air instead of in liquid helium, and we may therefore conclude that it is intimately connected with phenomena which occur in the neigh-

H. KAMERLINGH ONNES. "Further Experiments with Liquid Helium. F. Isotherms of Monatomic Gases etc. IX. Thermal Properties of Helium."

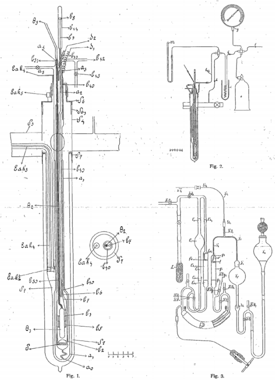


Fig. 1. Fig. 2. Fig. 3.

L'ARTICLE

Further experiments with Liquid Helium

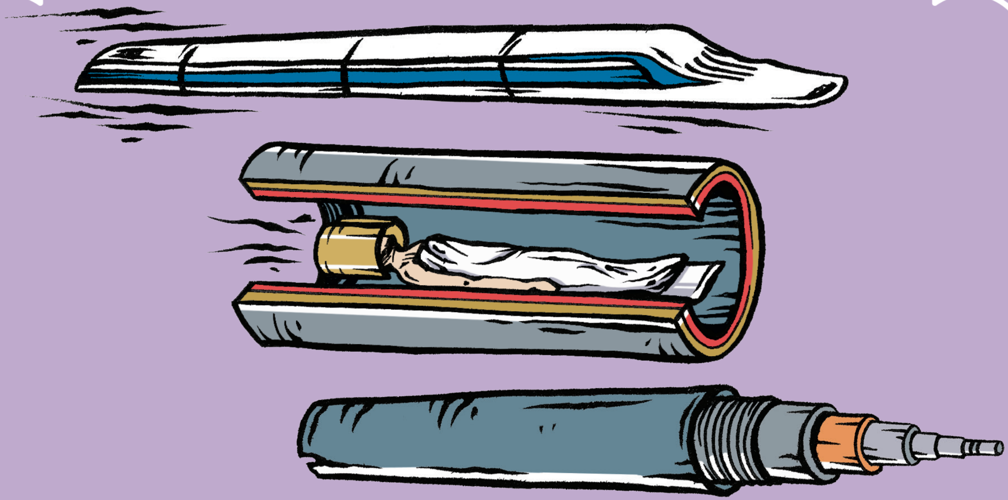
Com. N°124c from the Phys. Lab. at Leyden,1911

LA SUPRACONDUCTIVITÉ

K.ONNES, PRIX NOBEL, 1913

Pour ses recherches sur les propriétés de la matière aux basses températures,
qui conduisirent, entre autres, à la production d’hélium liquide.

LA SUPRACONDUCTIVITÉ



AUJOURD'HUI

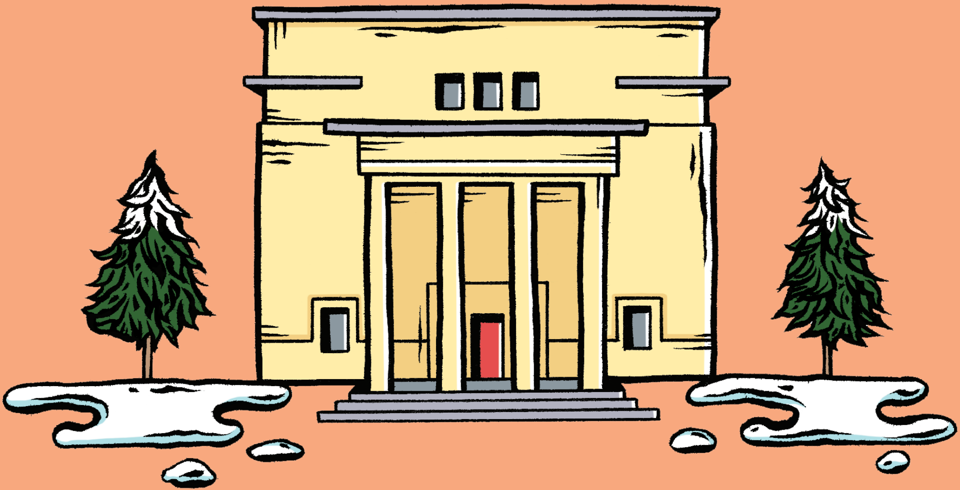
train à lévitation : le plus rapide au monde, imagerie médicale : par résonance magnétique, câbles électriques : conduisent mieux le courant

LA SUPRACONDUCTIVITÉ

LA SUPERFLUIDITÉ

— 1937 —

LA SUPERFLUIDITÉ

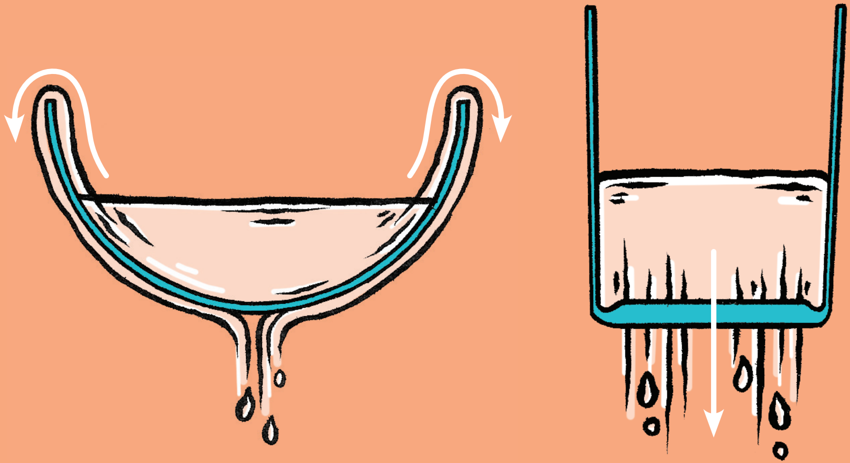


INSTITUT DES PROBLÈMES PHYSIQUES, MOSCOU, RUSSIE

LA SUPERFLUIDITÉ

P. KAPITSA

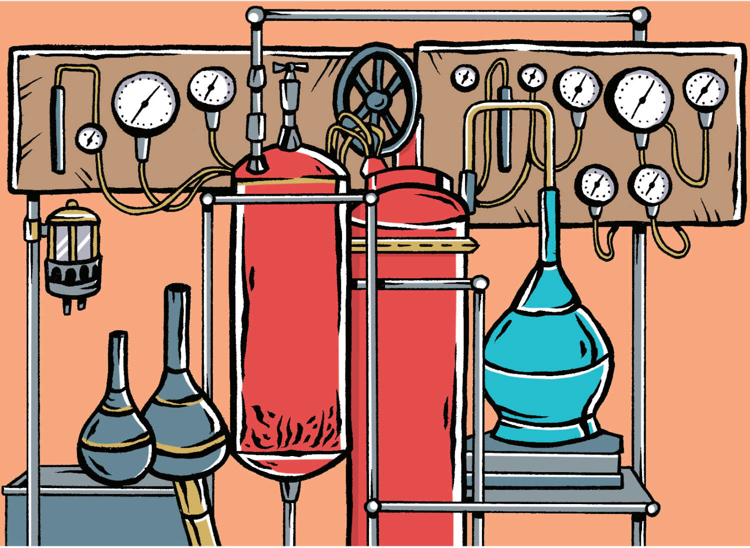
LA SUPERFLUIDITÉ



LA QUESTION

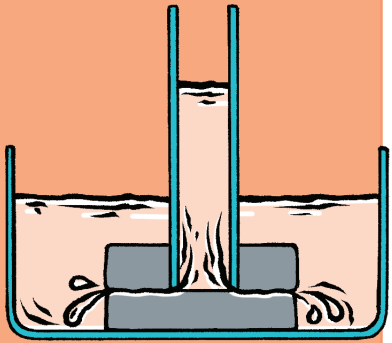
Que devient un liquide très près du zéro absolu s'il ne gèle pas ?

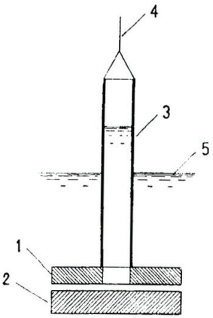
LA SUPERFLUIDITÉ



LE LABO

LA SUPERFLUIDITÉ



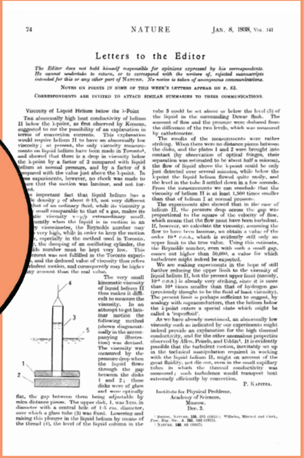


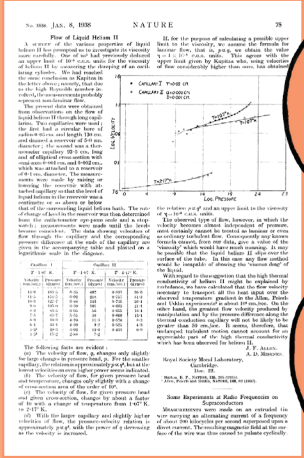
The very small kinematic viscosity of liquid helium II thus makes it difficult to measure the viscosity. In an attempt to get laminar motion the following method (shown diagramatically in the accompanying illustration) was devised. The viscosity was measured by the pressure drop when the liquid flows through the gap between the disks 1 and 2; these disks were of glass and were optically flat, the gap between them being adjustable by mica distance pieces. The upper disk, 1, was 3 cm. in diameter with a central hole of 1·5 cm. diameter, over which a glass tube (3) was fixed. Lowering and raising this plunger in the liquid helium by means of the thread (4), the level of the liquid column in the

LE RÉSULTAT

L'hélium est placé dans une colonne au dessus de deux plaques près du zéro absolu. Il arrive à s'écouler entre les plaques même quand elles se touchent ! Kapitsa appelle cela de la superfluidité.

LA SUPERFLUIDITÉ





LES ARTICLES

Viscosity of Liquid Helium below the λ-Point, P. Kapitsa, Nature 74, **141** (1938)

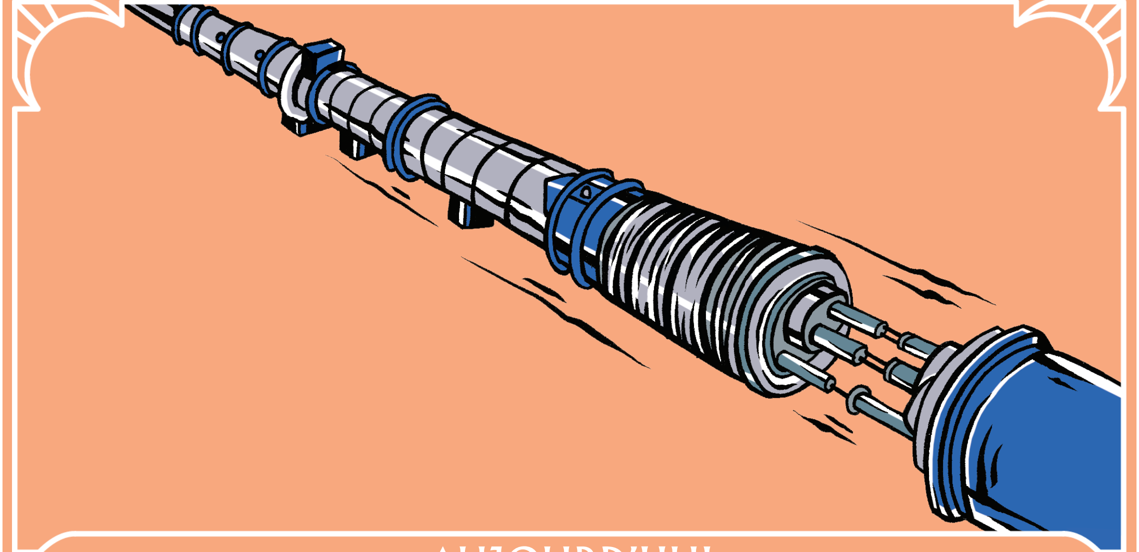
Flow of liquid helium II, J.F. Allen, A.D. Misener, Nature 75, **141** (1938)

LA SUPERFLUIDITÉ

P. KAPITSA, PRIX NOBEL, 1978

Pour ses inventions et découvertes fondamentales dans le domaine de la physique à basse température.

LA SUPERFLUIDITÉ



AUJOURD'HUI

L'hélium superfluide permet de refroidir les accélérateurs de particule comme le LHC.
Il est aussi l'outil indispensable pour faire des expériences de physique très près du zéro absolu.

LA SUPERFLUIDITÉ

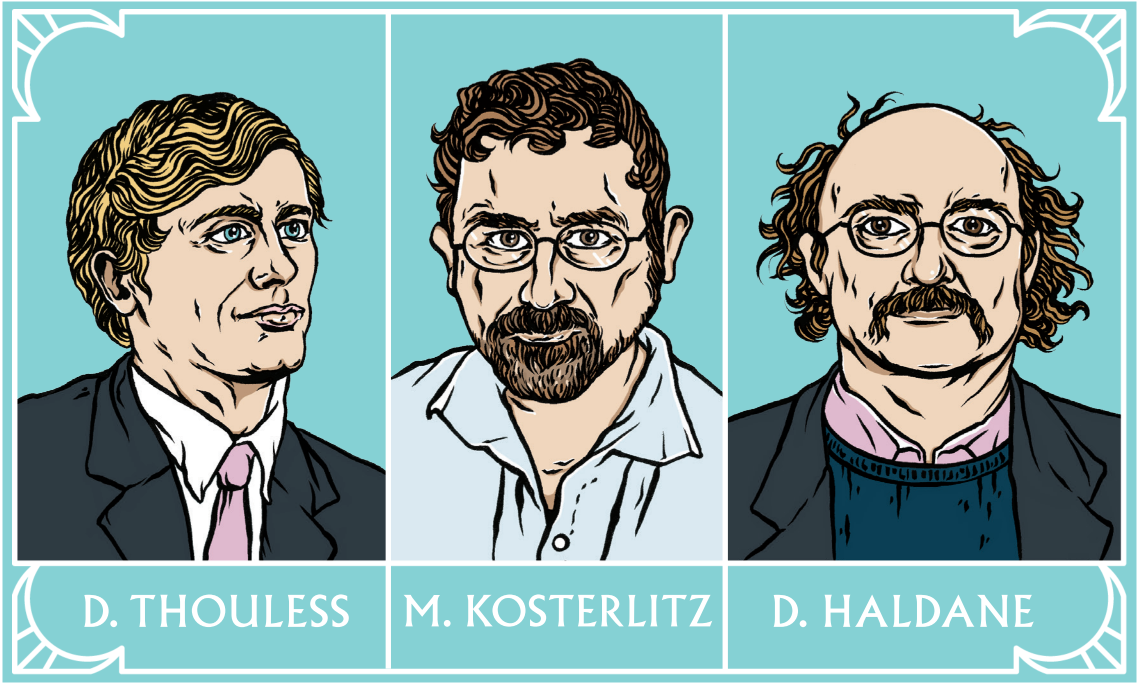
LA TOPOLOGIE

— 1972 — 1985 —

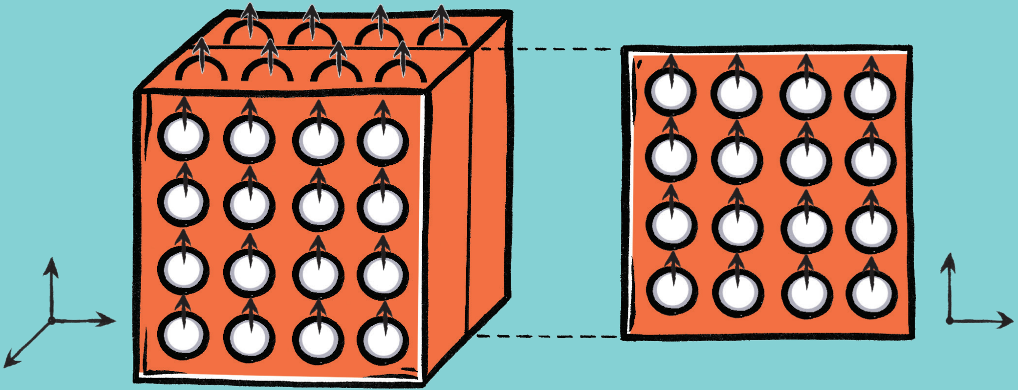
LA TOPOLOGIE


BIRMINGHAM UNIVERSITY, ANGLETERRE
UNIVERSITY OF SOUTHERN CALIFORNIA, USA

LA TOPOLOGIE


D. THOULESS
M. KOSTERLITZ
D. HALDANE

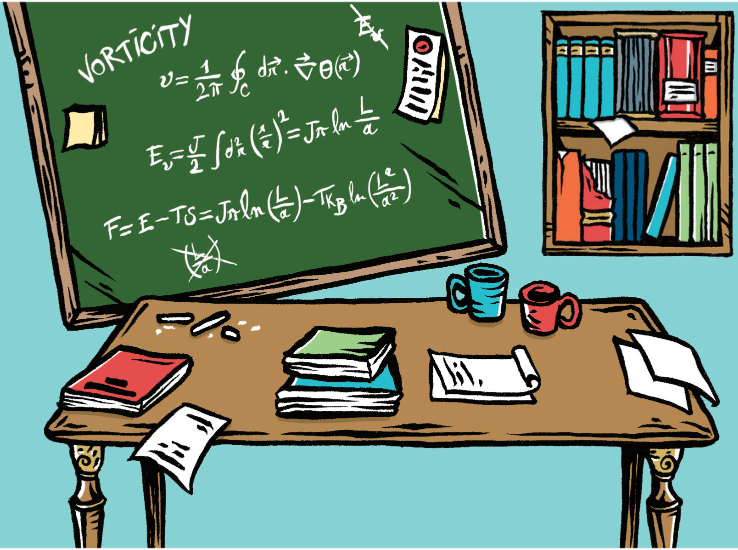
LA TOPOLOGIE



LA QUESTION

Un supraconducteur ou un aimant peuvent-ils exister à deux dimensions ?

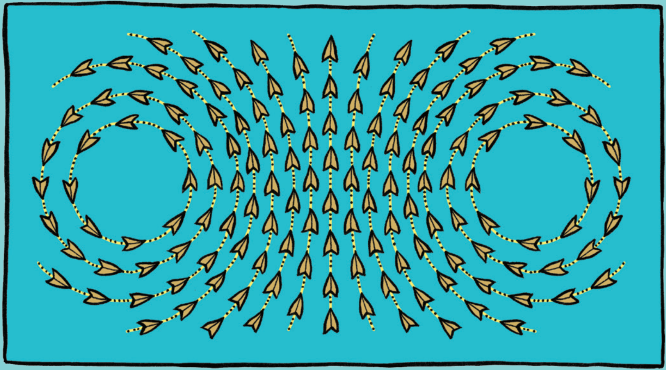
LA TOPOLOGIE



LE LABO

LA TOPOLOGIE

$$\frac{\pi J}{k_B T_c} - 1 \approx \pi \, \tilde{y}_c(0) \exp\left(\frac{-\pi^2 J}{k_B T_c}\right) \approx 0{\cdot}12 \, .$$



LE RÉSULTAT

De nouveaux états peuvent apparaître dans la matière pour des raisons « topologiques ». Par exemple dans des aimants ou des suprafluides à 2 dimensions, il apparaît des vortex et anti-vortex qui permettent à l'ordre de se maintenir.

LA TOPOLOGIE

J. Phys. C: Solid State Phys., Vol. 6, 1973. Printed in Great Britain. © 1973

Ordering, metastability and phase transitions in two-dimensional systems

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Received 13 November 1972

Abstract. A new definition of order called topological order is proposed for two-dimensional systems in which no long-range order of the conventional type exists. The possibility of a phase transition characterized by a change in the response of the system to an external perturbation is discussed in the context of a mean field type of approximation. The critical behaviour found in this model displays very weak singularities. The application of these ideas to the *xy* model of magnetism, the solid–liquid transition, and the neutral superfluid are discussed. This type of phase transition cannot occur in a superconductor nor in a Heisenberg ferromagnet, for reasons that are given.

1. Introduction

Peierls (1935) has argued that thermal motion of long-wavelength phonons will destroy the long-range order of a two-dimensional solid in the sense that the mean square deviation of an atom from its equilibrium position increases logarithmically with the size of the system, and the Bragg peaks of the diffraction pattern formed by the system are broad instead of sharp. The absence of long-range order of this simple form has been shown by Mermin (1968) using rigorous inequalities. Similar arguments can be used to show that there is no spontaneous magnetization in a two-dimensional magnet with spins with more than one degree of freedom (Mermin and Wagner 1966) and that the expectation value of the superfluid order parameter in a two-dimensional Bose fluid is zero (Hohenberg 1967).

On the other hand there is inconclusive evidence from the numerical work on a two-dimensional system of hard discs by Alder and Wainwright (1962) of a phase transition between a gaseous and solid state. Stanley and Kaplan (1966) found that high-temperature series expansions for two-dimensional spin models indicated a phase transition in which the susceptibility becomes infinite. The evidence for such a transition is much stronger for the *xy* model (spins confined to a plane) than for the Heisenberg model, as can be seen from the papers of Stanley (1968) and Moore (1969). Low-temperature expansions obtained by Wegner (1967) and Berezinskii (1970) give a magnetization proportional to some power of the field between zero and unity, and indicate the possibility of a sharp transition between such behaviour and the high-temperature régime where the magnetization is proportional to the applied field.

In this paper we present arguments in favour of a quite different definition of long-range order which is based on the overall properties of the system rather than on the

To conclude this section on the model system, we would like to point out that the assumption of a very dilute system ($e^{-2\beta\mu} \ll 1$) is not necessarily valid in a real system. However, we expect that the qualitative arguments will go through even in such a case and the general form of the results will be unchanged. We can imagine increasing the cutoff r_0 to some value R_0 such that the energy of two charges a distance R_0 apart is $2\mu(R_0)$ where $\exp\{-2\mu(R_0)\beta\} \ll 1$. For charges further apart than R_0, we can use the theory as outlined previously. The boundary conditions given by equation (20) will be changed to

$$y(0) = \frac{2q^2}{k_B T\epsilon(R_0)} - 4 \tag{41}$$

with $\epsilon(R_0)$ an unknown function. The critical temperature and the dielectric constant will now be determined in terms of $\epsilon(R_0)$ and $\mu(R_0)$. To determine these two quantities, a more sophisticated treatment is required, but we expect that the behaviour of the dielectric constant and specific heat at the critical temperature will be unchanged.

3. The two-dimensional *xy* model

The two-dimensional *xy* model is a system of spins constrained to rotate in the plane of the lattice which, for simplicity, we take to be a simple square lattice with spacing $\boldsymbol{a}$. The hamiltonian of the system is

$$H = -J\sum_{\langle ij\rangle} \boldsymbol{S}_i . \boldsymbol{S}_j = -J\sum_{\langle ij\rangle} \cos(\phi_i - \phi_j) \tag{42}$$

where $J > 0$ and the sum $\langle ij\rangle$ over lattice sites is over nearest neighbours only. We have taken $|\boldsymbol{S}_i| = 1$ and ϕ_i is the angle the ith spin makes with some arbitrary axis. Only slowly varying configurations, that is, those with adjacent angles nearly equal, will give any significant contribution to the partition function so that may expand the hamiltonian up to terms quadratic in the angles.

It has been shown by many authors (Mermin and Wagner 1966, Wegner 1967, Berezinskii 1970) that this system does not have any long-range order as the ground state is unstable against low-energy spin-wave excitations. However, there is some evidence (Stanley 1968, Moore 1969) that this system has a phase transition, but it cannot be of the usual type with finite mean magnetization below T_c. As we shall show, there exist metastable states corresponding to vortices which are closely bound in pairs below some critical temperature, while above this they become free. The transition is characterized by a sudden change in the response to an applied magnetic field.

Expanding about a local minimum of H

$$H - E_0 \approx \tfrac{1}{2}J\sum_{\langle ij\rangle}(\phi_i - \phi_j)^2 = J\sum_{\boldsymbol{r}}(\Delta\phi(\boldsymbol{r}))^2 \tag{43}$$

where Δ denotes the first difference operator, $\phi(\boldsymbol{r})$ is a function defined over the lattice sites, and the sum is taken over all the sites. If we consider the system in the configuration of figure 1, its energy is, from equation (43)

$$H - E_0 \approx \pi J \ln\frac{R}{a} \tag{44}$$

where R is the radius of the system. Thus we have a slowly varying configuration, which we shall call a vortex, whose energy increases logarithmically with the size of the system.

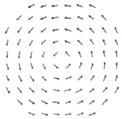


Figure 1. An isolated vortex in the *xy* model.

From the arguments of the Introduction, this suggests that a suitable description of the system is to approximate the hamiltonian by terms quadratic in $\Delta\phi(\boldsymbol{r})$ and split this up into a term corresponding to the vortices and another to the low-energy excitations (spin waves).

We extend the domain of $\phi(\boldsymbol{r})$ to $-\infty < \phi(\boldsymbol{r}) < \infty$ to allow for the fact that, in the absence of vortices, $\langle(\phi(\boldsymbol{r}) - \phi(\boldsymbol{r}'))^2\rangle$ increases like $\ln(|\boldsymbol{r} - \boldsymbol{r}'|)$ (Berenzinskii 1971). Thus, at large separations, the spins will have gone through several revolutions relative to one another. If we now consider a vortex configuration of the type of figure 1, as we go round some closed path containing the centre of the vortex, $\phi(\boldsymbol{r})$ will change by 2π for each revolution. Thus, for a configuration with no vortices, the function $\phi(\boldsymbol{r})$ will be single-valued, while for one with vortices it will be many-valued. This may be summarized by

$$\oint\!\!\!\!\sum \Delta\phi(\boldsymbol{r}) = 2\pi q \qquad q = 0, \pm 1, \pm 2 \ldots \tag{45}$$

where the sum is over some closed contour on the lattice and the number q defines the total strength of the vortex distribution contained in the contour. If a single vortex of the type shown in figure 1 is contained in the contour, then $q = 1$.

Let now $\phi(\boldsymbol{r}) = \psi(\boldsymbol{r}) + \bar{\phi}(\boldsymbol{r})$, where $\bar{\phi}(\boldsymbol{r})$ defines the angular distribution of the spins in the configuration of the local minimum, and $\psi(\boldsymbol{r})$ the deviation from this. The energy of the system is now

$$H - E_0 \approx J\sum_{\boldsymbol{r}}(\Delta\psi(\boldsymbol{r}))^2 + J\sum_{\boldsymbol{r}}(\Delta\bar{\phi}(\boldsymbol{r}))^2 \tag{46}$$

where

$$\oint\!\!\!\!\sum \Delta\psi(\boldsymbol{r}) = 0 \qquad \text{and} \qquad \oint\!\!\!\!\sum \Delta\bar{\phi}(\boldsymbol{r}) = 2\pi q. \tag{47}$$

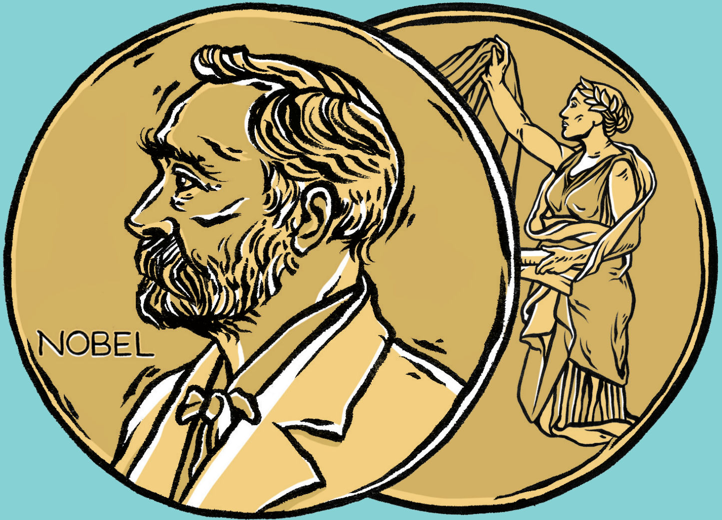
The cross term vanishes because of the condition (47) obeyed by $\psi(\boldsymbol{r})$. Clearly the configuration of absolute minimum energy corresponds to $q = 0$ for every possible contour when $\bar{\phi}(\boldsymbol{r})$ is the same for all lattice sites. We see from equation (45) that, if we shrink the contour so that it passes through only four sites as in figure 2, we will obtain the strength

L'ARTICLE

Ordering, metastability and phase transitions in two-dimensional systems

J.M. Kosterlitz, D.J. Thouless, Journal of Physics C: Solid State Physics, 6, 1181 (1973).

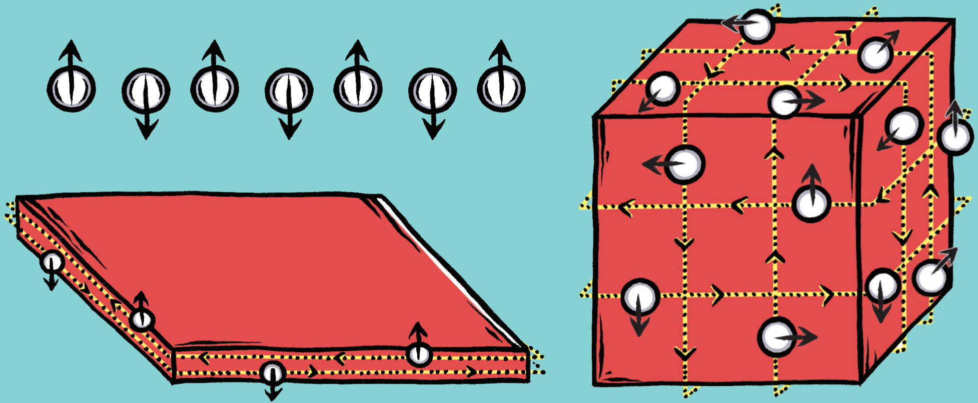
LA TOPOLOGIE



D. THOULESS, M. KOSTERLITZ, D. HALDANE, PRIX NOBEL, 2016

Pour des découvertes théoriques de phases et transitions topologiques dans la matière.

LA TOPOLOGIE



AUJOURD'HUI

Ces travaux ont permis de découvrir un grand nombre de nouveaux états topologiques à une, deux et trois dimensions dans des aimants, des métaux, ou des isolants.

LA TOPOLOGIE